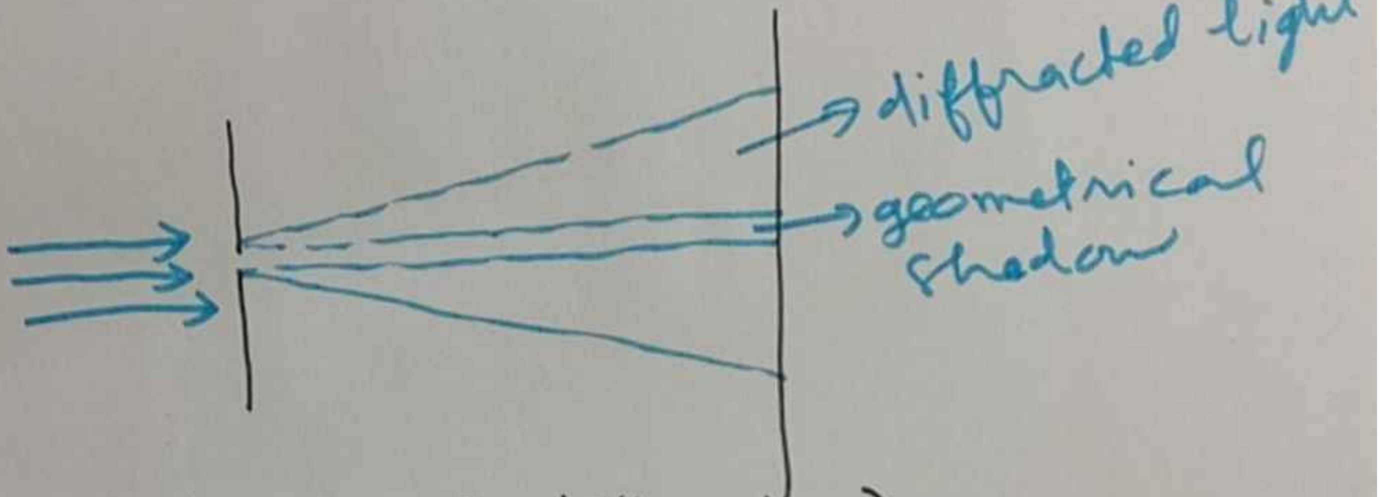
Diffraction of light

Fresnel's half Period Zones



(Fresnel diffraction)

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Diffraction of Light

For diffraction, wavelength of the wave should be comparable to the size of the obstacle.

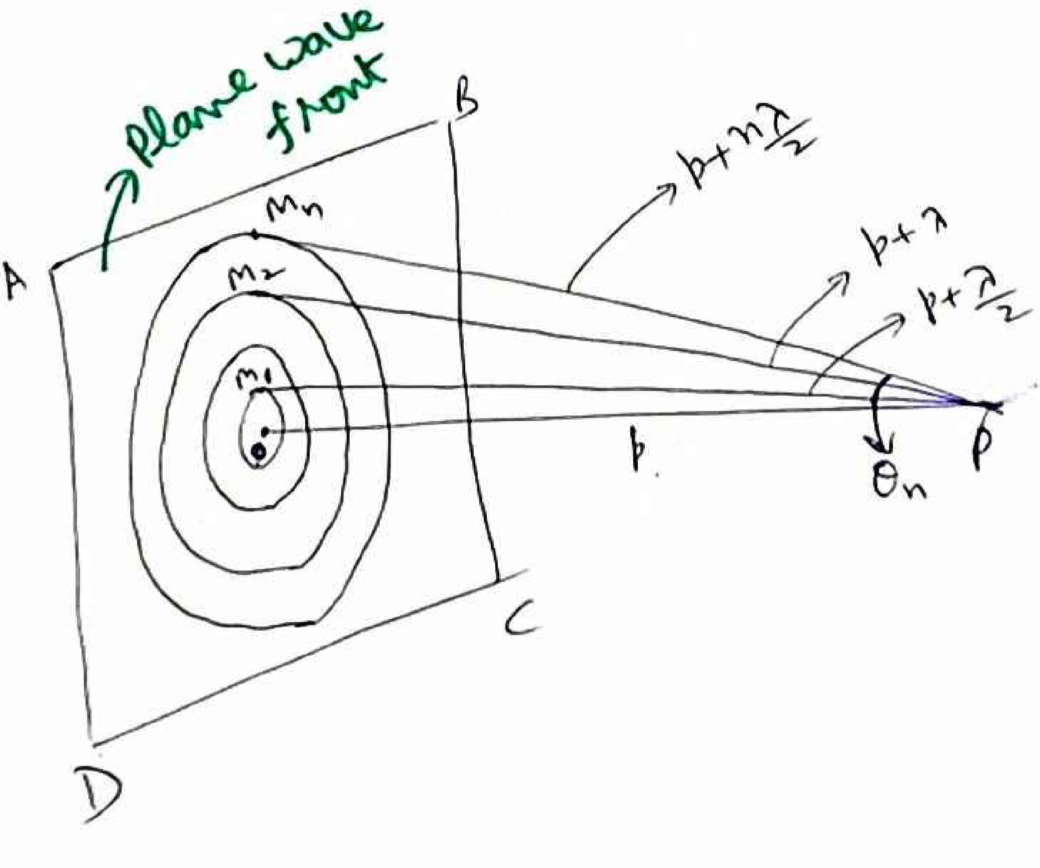
Fresnel diffraction

Source and Screen are at the finite distance from the obstacle. In this case no lenses are used and the incident wavefront is either spherical or cylinderical.

Fraunhofer diff.

Source and Screen are effectively at infinite distance from the obstacle. This is done by placing the source and Screen in the focal plane of two lenses. In this case incident wavefront is plane. Fraunhofer diff. is a limiting case of more general Fresnel diff..

Fresnel's half Period Zones-



Diffraction of light

Let ABCD a plane wavefront. Let P be the external point at which effect of the entire wavefront is to be estimated.

Let $PO = p$

then taking P as centre draw spheres of radii $p+\frac{\lambda}{2}$, $p+\lambda$, $p+\frac{3\lambda}{2}$, $p+2\lambda$, ---.

Intersections of these spheres with plane ABCD will be the concentric circles with centre O.

Area of the innermost circle is called first half period zone.

Area between the first and second circle is called second half period zone and so on.

Area of a zone —

Area of n^{th} zone $= \pi OM_n^2 - \pi OM_{n-1}^2$

$$= \pi\left[(PM_n^2 - PO^2) - (PM_{n-1}^2 - PO^2)\right]$$

$$= \pi\left[\left\{\left(p+\frac{n\lambda}{2}\right)^2 - p^2\right\} - \left\{\left(p+(n-1)\frac{\lambda}{2}\right)^2 - p^2\right\}\right]$$

$$= \pi\left[p^2 + \frac{n^2\lambda^2}{4} + pn\lambda - p^2 - p^2 - \frac{(n-1)^2\lambda^2}{4} - (n-1)p\lambda + p^2\right]$$

$$= \pi\left[p\lambda + \frac{n^2\lambda^2}{4} - \frac{(n-1)^2\lambda^2}{4}\right]$$

$$= \pi\left[p\lambda + \frac{\lambda^2}{4}(2n-1)\right] = \pi p\lambda$$

($\because p >> \lambda^2$ therefore λ^2 term is neglected)

$\Rightarrow$ area of the zone is approximately same for each zone (but actually it increases slightly with n).

Average distance of each zone from P –

average distance of n^{th} zone from P

$$= \frac{1}{2}\left[\left\{\left(p+\frac{n\lambda}{2}\right)\right\} + \left\{p + \frac{(n-1)\lambda}{2}\right\}\right]$$

$$= p + \frac{(2n-1)\lambda}{4}$$

Amplitude at P due to a zone –

Amplitude at P due to a zone is

(a) directly proportional to the area of the zone

(b) Inversely proportional to the average distance of the zone from P

(c) directly proportional to the obliquity factor $(1+\cos\theta_n)$.

Thus amplitude at P due to n^{th} zone will be

$$R_n \propto \frac{\pi\left[p\lambda + \frac{\lambda^2}{4}(2n-1)\right]}{p+(2n-1)\frac{\lambda}{4}} (1+\cos\theta_n)$$

$$\propto \pi\lambda(1+\cos\theta_n) ..$$

as n increases, θ_n increases, therefore $\cos\theta_n$ decreases. Therefore R_n decreases with increasing n.

Resultant amplitude at P due to all zones –

let $R_1, R_2, R_3 --- R_n$ be the amplitude at P due to I, II, --- n^{th} zones respectively.

clearly, $R_1 > R_2 > R_3 > R_4 --- > R_n$

Also, since the average distance of P from consecutive zones differ by $\frac{\lambda}{2}$, the ~~consequence~~ waves from two consecutive zones reach P in opposite phase. Therefore if R_1 is +ive then R_2 will be negative, and so on.

∴ Resultant amplitude of P due to whole wave front

$$R = R_1 - R_2 + R_3 - R_4 + R_5 - --- + (-1)^{n-1} R_n$$

Now, since successive terms $R_1, R_2, R_3 ---$ gradually decrease in magnitude, therefore we can write,

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$$R_2 = \frac{R_1 + R_3}{2}, \quad R_4 = \frac{R_3 + R_5}{2}, \text{ etc.}$$

$$\therefore R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right) + \cdots$$

the last term being $\frac{R_n}{2}$ (if n is odd)

or $\left(\frac{R_{n-1}}{2} - R_n\right)$ (if n is even)

Thus $R = \frac{R_1}{2} + \frac{R_n}{2}$ (n being odd)

or $R = \frac{R_1}{2} + \frac{R_{n-1}}{2} - R_n$ (if n is even)

In practice, n is very large

$\therefore$ we can write $R_n = R_{n-1} = 0$

$$\boxed{\therefore R = \frac{R_1}{2}}$$

$\Rightarrow$ amplitude at P due to complete wave front is just half that due to the first half period zone alone.